At Centenary of Einstein's General Theory of Relativity: Testing the Nature of Accreting Black Holes

Selected topics as science drivers for mission proposals

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Objects and models

- Active galactic nuclei
- Stellar-mass black holes
- Intermediate-mass black holes (?)

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- Accretion disc
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  - ...time-dependent, non-axisymmetric

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- GR effects taken into account
- Link to a spectrum-fitting procedure $\rightarrow$ parameters

El.-vac. soln. w/rotating BH

Exact solution: magnetized Kerr-Newman BH:

$$
\begin{aligned}
g &= |\Lambda|^2\Sigma\left(\Delta^{-1}\,\mathrm{d}r^2 + \,\mathrm{d}\theta^2 - \Delta A^{-1}\,\mathrm{d}t^2\right) \\
&\quad +|\Lambda|^{-2}\Sigma^{-1}A\sin^2\theta\,(\,\mathrm{d}\phi - \omega\,\mathrm{d}t)^2\,,
\end{aligned}
$$

$\Sigma = r^2 + a^2\cos^2\theta$, $\Delta = r^2 - 2Mr + a^2 + e^2$,
$A = (r^2 + a^2)^2 - \Delta a^2\sin^2\theta$ are functions from the Kerr-Newman metric.
$\Lambda = 1 + \beta\Phi - \frac{1}{4}\beta^2\mathcal{E}$ is given in terms of the Ernst complex potentials $\Phi(r,\theta)$ and $\mathcal{E}(r,\theta)$:

$$
\begin{aligned}
\Sigma\Phi &= ear\sin^2\theta - \Im e\left(r^2 + a^2\right)\cos\theta, \\
\Sigma\mathcal{E} &= -A\sin^2\theta - e^2\left(a^2 + r^2\cos^2\theta\right) \\
&\quad +2\Im a\left[\Sigma\left(3 - \cos^2\theta\right) + a^2\sin^4\theta - re^2\sin^2\theta\right]\cos\theta.
\end{aligned}
$$

Bretón Baez N., García Díaz A., J. Math. Phys. **27**, 562 (1986)

Thank you!

<u>See Additional Material for Discussion:</u>

High-frequency elmg. waves

Polarization tensor

Stokes parameters

High-frequency elmg. waves

Basic equations – vacuum case: $F^{\mu\nu}{}_{;\nu} = 0, \qquad {}^{\star}F^{\mu\nu}{}_{;\nu} = 0.$

$$E^{\alpha} = F^{\alpha\beta} u_{\beta}, \; {}^{\star}F_{\mu\nu} \equiv \tfrac{1}{2}\varepsilon_{\mu\nu}{}^{\rho\sigma} F_{\rho\sigma}$$

An electromagnetic wave is an approximate test-field solution of the Maxwell equations:

$$F_{\alpha\beta} = \Re e \left[u_{\alpha\beta} \, e^{\Im S(x)} \right].$$

A fixed background geometry is asssumed.

- Phase $S(x)$ ... rapidly varying function
- Amplitude $u_{\alpha\beta}$ ... slowly varying function
- Wave vector $k_{\alpha} \equiv S_{,\alpha}$ ... paralel transport, null geodesics

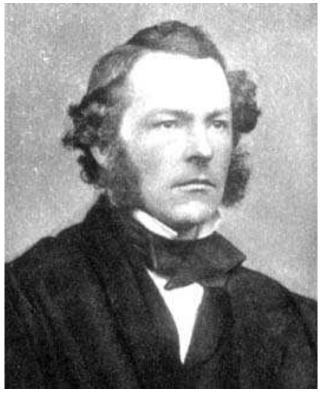
$$k_{\alpha;\beta} \, k^{\beta} = 0, \quad k_{\alpha} k^{\alpha} = 0.$$

Polarization tensor

- Polarization tensor ... $J_{\alpha\beta\gamma\delta} \equiv \frac{1}{2}\langle F_{\alpha\beta}\bar{F}_{\gamma\delta}\rangle$
- In observer's rest-frame ... $J_{\alpha\beta} \equiv J_{\alpha\beta\gamma\delta}\, u^{\beta}\, u^{\delta} = \langle E_{\alpha}\bar{E}_{\beta}\rangle$
- Four parameters ... $S_{A} \equiv \frac{1}{2}(k_{\alpha}u^{\alpha})^2 F_{A} \quad (A = 0, \ldots, 3)$

(F_A ... constructed by projecting onto a tetrad $e^{\alpha}_{(i)}$)

"On the composition and resolution of streams of polarized light from different sources"



References: [1] Sir George Stokes (1852), Trans. Cambridge Phil. Soc., 9, 399
[2] Chandrasekhar (1950), *Radiative Transfer* (Oxford: Clarendon)
[3] Cocke & Holm (1972), Nature, 240, 161
[4] Jauch & Rohrlich (1955), *The Theory of Photons and Electrons* (Reading: Wesley)

Stokes parameters

$$
\begin{aligned}
S_0 &\equiv J_{\alpha\beta}\left(e^{\alpha}_{(1)}e^{\beta}_{(1)} + e^{\alpha}_{(2)}e^{\beta}_{(2)}\right) = \langle |E_{(1)}|^2 + |E_{(2)}|^2 \rangle \\
S_1 &\equiv J_{\alpha\beta}\left(e^{\alpha}_{(1)}e^{\beta}_{(1)} - e^{\alpha}_{(2)}e^{\beta}_{(2)}\right) = \langle |E_{(1)}|^2 - |E_{(2)}|^2 \rangle \\
S_2 &\equiv J_{\alpha\beta}\left(e^{\alpha}_{(1)}e^{\beta}_{(2)} + e^{\alpha}_{(2)}e^{\beta}_{(1)}\right) = \langle E_{(1)}\bar{E}_{(2)} + E_{(2)}\bar{E}_{(1)} \rangle \\
S_3 &\equiv \Im J_{\alpha\beta}\left(e^{\alpha}_{(1)}e^{\beta}_{(2)} - e^{\alpha}_{(2)}e^{\beta}_{(1)}\right) = \Im\langle E_{(1)}\bar{E}_{(2)} - E_{(2)}\bar{E}_{(1)} \rangle
\end{aligned}
$$

S_1, S_2, and S_3 determine the polarization state.

References: [5] Anile (1989), *Relativistic fluids and magneto-fluids* (Cambridge)
[6] Madore (1974), Comm. Math. Phys., 38, 103
[7] Bičák & Hadrava (1975), A&A, 44, 389
[8] Breuer & Ehlers (1980), Proc. Roy. Soc. Lond. A, 370, 389
[9] Broderick & Blandford (2003), MNRAS, 342, 1280